Dr. Marques Sophie Linear Algebra section 008 Fall Semester 2015
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Quiz #4

Justify all your answers completely (Or with a proof or with a counter example) unless mentioned differently. No step should be a mystery or bring a question. The grader cannot be expected to work his way through a sprawling mess of identities presented without a coherent narrative through line. If he can't make sense of it in finite time you could lose serious points. Coherent, readable exposition of your work is half the job in mathematics. You will loose serious points if your exposition is messy, incomplete, uses mathematical symbols not adapted...

Problems:

1. Give the definition of an inner product $<,>$ for a vector space V and the definition of the norm relative to $<,>$.
2. Using only the axioms of the inner product and the definition of the norm, prove that $||u+v||^2 + ||u-v||^2 = 2||u||^2 + 2||v||^2$. (Justify all your steps)
3. Compute the orthogonal projection of $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ onto the line through $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and the origin and give the closest point of this line to $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. (Justify all your answers.)
4. Use the Gram-Schmidt process to produce an orthogonal basis for W the subspace generated by the columns of A where

$$A = \begin{pmatrix} 3 & 8 \\ 0 & 5 \\ -1 & -6 \end{pmatrix}$$

   (Show your work)
5. Give a QR factorization of A of the previous question. (Justify)